

# The nucleon–nucleon interaction and properties of the nucleon in a $\pi\rho\omega$ soliton model including a dilaton field with anomalous dimension

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## Abstract

We investigate an extended chiral soliton model which includes  $\pi, \rho, \omega$  and  $\sigma$  mesons as explicit degrees of freedom. The Lagrangian incorporates chiral symmetry and broken scale invariance. A scalar–isoscalar meson  $\sigma$  is associated with a quarkonium dilaton field with a mass  $m_\sigma \approx 550 \text{ MeV}$ . We show that the scalar field with anomalous dimension slightly changes the static and electromagnetic properties of the nucleon. In contrast, it plays a significant role in nucleon–nucleon dynamics and gives an opportunity to describe well the two–nucleon interaction.

Keywords: [ Skyrme model, sigma meson, scale dimension, nucleon properties, meson-nucleon form factors, nucleon-nucleon interaction.]

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# 1 Introduction

In ref.[1] Furnstahl, Tang and Serot (FTS) proposed a new model for nuclear matter and finite nuclei that realizes QCD symmetries such as chiral symmetry, broken scale invariance and the phenomenology of vector meson dominance. An important feature of this approach is the inclusion of light scalar degrees of freedom, which are given an anomalous scale dimension. The vacuum dynamics of QCD is constrained by the trace anomaly and related low-energy theorems of QCD. The scalar-isoscalar sector of the theory is divided into a low mass part that is adequately described by a scalar meson (quarkonium) with anomalous dimension and a high mass part (gluonium), that can be “integrated out”, leading to various couplings among the remaining fields. The application of the model to the properties of nuclear matter as well as finite nuclei gave a satisfactory description. Further developments of the model [2, 3] showed that the light scalar related to the trace anomaly can play a significant role not only in the description of bound nucleons but also in the description of heavy-ion collisions. It was also shown that the anomalous cannot be due to an effect of nuclear density on the trace anomaly of QCD [2].

Here a natural question arises: What is the role of this light quarkonium in the description of the properties of a single nucleon, when it is taken into account in topological nonlinear chiral soliton models, which are similar to the FTS effective Lagrangian on the single nucleon level? In the present paper we introduce a dilaton field with an anomalous dimension into the  $\pi\rho\omega$ -model [4] and investigate some properties of single nucleons which emerge as solitons in the sector with baryon number one ( $B = 1$ ).

It is well known that a scalar-isoscalar meson,<sup>4</sup> the sigma, plays an important role in the nucleon-nucleon (NN) interaction especially within one-boson-exchange (OBE) models [6]. We remark that the missing medium range attraction was a long standing puzzle in Skyrme like models. Lately it has been shown that [7] explicit inclusion of a scalar meson into the Skyrme model produces in a natural way the desired attraction. But these studies have two shortcomings. Firstly, the scalar meson used in such models [7] has nothing to do with OBE phenomenology, as it has a large mass and is identified with a gluonium state. Secondly, the Lagrangians used in refs. [7] do not include explicit omega mesons at all, which may “spoil” the mechanism of the attraction due to its strong repulsion.<sup>5</sup> Therefore, it would be quite interesting to investigate the central part of the NN interaction when the light scalar ( $\sigma$ ) and  $\omega$ -mesons are both taken into account explicitly. This is what is done here. In particular, it is important that if one is to properly describe the intermediate range attraction in the central NN interaction, the successful description of the single nucleon properties within the  $\pi\rho\omega$  model should not be destroyed. We also note that in an soliton approach with explicit regulated two-pion loop graphs one is able to get the proper intermediate range attraction. In that case, however, one does not stay within a simple OBE approach any more (as done here) and also needs to calculate the modifications of the isovector two-pion exchange to the  $\rho$  and so on. For comparison, we mention that recent developments of the original Bonn OBE potential performed at Jülich also include multi-meson exchanges leading to a renormalization of various interactions, couplings and cut-off parameters [9]. The model we investigate is related closely

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<sup>4</sup>Here and in what follows, we call it sigma meson for simplicity. Although we assume it to be some kind of quarkonium state, its precise dynamical nature is of no direct relevance for the following arguments.

<sup>5</sup>Of course, the stabilization of the soliton in refs.[7] via higher derivative terms also leads to repulsion in the central NN interaction, but this is less easily be interpreted as single meson exchange.

to the OBE approximation of the NN force.

## 2 The $\pi\rho\omega\sigma$ model

Including a  $\sigma$ -meson by means of the scale invariance and trace anomaly of QCD into the  $\pi\rho\omega$ -model [8] can be done in terms of the following chiral Lagrangian of the coupled  $\pi\rho\omega\sigma$  system,

$$\begin{aligned}\mathcal{L} = & \frac{S_0^2 e^{-2\sigma/d}}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{f_\pi^2 e^{-2\sigma/d}}{4} \text{Tr} L_\mu L^\mu - \frac{f_\pi^2 e^{-2\sigma/d}}{2} \text{Tr} [l_\mu + r_\mu + ig \vec{\tau} \vec{\rho}_\mu + ig \omega_\mu]^2 + \\ & + \frac{3}{2} g \omega_\mu B^\mu - \frac{1}{4} (\omega_{\mu\nu} \omega^{\mu\nu} + \vec{\rho}_{\mu\nu} \vec{\rho}^{\mu\nu}) + \frac{f_\pi^2 m_\pi^2 e^{-3\sigma/d}}{2} \text{Tr} (U - 1) - \\ & - \frac{d^2 S_0^2 m_\sigma^2}{16} \left[ 1 - e^{-4\sigma/d} \left( \frac{4\sigma}{d} + 1 \right) \right],\end{aligned}\tag{1}$$

where the pion fields are parametrized in terms of  $U = \exp(i\vec{\tau} \cdot \vec{\pi}/f_\pi)$  and  $\xi = \sqrt{U}$ , left/right-handed currents are given by  $L_\mu = U^\dagger \partial_\mu U$ ,  $l_\mu = \xi^\dagger \partial_\mu \xi$ ,  $r_\mu = \xi \partial_\mu \xi^\dagger$ , and the pertinent vector meson ( $\vec{\rho}, \omega$ ) field strength tensors are  $\vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu + g[\vec{\rho}_\mu \times \vec{\rho}_\nu]$  and  $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ . Furthermore, the topological baryon number current is given by  $B^\mu = \varepsilon^{\mu\alpha\beta\gamma} \text{Tr} L_\alpha L_\beta L_\gamma / (24\pi^2)$ .

In Eq.(1),  $S_0$  is the vacuum expectation value of scalar field in free space matter,  $f_\pi$  is the pion decay constant ( $f_\pi = 93$  MeV) and  $g = g_{\rho\pi\pi}$  is determined through the KSFR relation  $g = m/\sqrt{2}f_\pi$ . The model assumes the masses of  $\rho$  and  $\omega$  mesons to be equal,  $m_\rho = m_\omega = m$ . The mass of the  $\sigma$  is related to the gluon condensate in the usual way [1, 10]  $m_\sigma = 2\sqrt{C_g}/(dS_0)$ , where  $d$  is the scale dimension of scalar field ( $d > 1$ ). Being “mapped” onto the states of a nucleon, the Lagrangian Eq.(1) will be similar to the FTS effective Lagrangian.

Nucleons arise as soliton solutions from the Lagrangian Eq.(1) in the sector with baryon number  $B = 1$ . To construct them one goes through a two step procedure. First, one finds the classical soliton which has neither good spin nor good isospin. Then an adiabatic rotation of the soliton is performed and it is quantized collectively.<sup>6</sup> The classical soliton follows from Eq.(1) by virtue of a spherical symmetrical ansätze for the meson fields:

$$U(\vec{r}) = \exp(i\vec{\tau} \hat{r} \Theta(r)), \quad \rho_i^a = \varepsilon_{iak} \hat{r}_k \frac{G(r)}{gr}, \quad \omega_\mu(\vec{r}) = \omega(r) \delta_{\mu 0}, \quad \sigma(\vec{r}) = \sigma(r). \tag{2}$$

In what follows we call  $\Theta(r)$ ,  $G(r)$ ,  $\omega(r)$ , and  $\sigma(r)$  the pion-,  $\rho$ -,  $\omega$ -, and  $\sigma$ -meson profile functions, respectively. The pertinent boundary conditions to ensure baryon number one and finite energy are,  $\Theta(0) = \pi$ ,  $G(0) = -2$ ,  $\omega'(0) = \sigma'(0) = 0$ ,  $\Theta(\infty) = G(\infty) = \omega(\infty) = \sigma(\infty) = 0$ . To project out baryonic states of good spin and isospin, we perform a time-independent SU(2)

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<sup>6</sup>We again refer the reader to ref. [8] for details.

rotation

$$\begin{aligned}
U(\vec{r}, t) &= A(t)U(\vec{r})A^+(t), \quad \xi(\vec{r}, t) = A(t)\xi(\vec{r})A^+(t) \\
\sigma(\vec{r}, t) &= \sigma(r), \quad \omega(\vec{r}, t) = \frac{\phi(r)}{r}[\vec{K}\hat{r}] \\
\vec{\tau} \cdot \vec{\rho}_0(\vec{r}, t) &= \frac{2}{g}A(t)\vec{\tau} \cdot (\vec{K}\xi_1(r) + \hat{r}\vec{K} \cdot \hat{r}\xi_2(r))A^+(t), \\
\vec{\tau} \cdot \vec{\rho}_i(\vec{r}, t) &= A(t)\vec{\tau} \cdot \vec{\rho}_i(\vec{r})A^+(t)
\end{aligned} \tag{3}$$

with  $2\vec{K}$  the angular frequency of the spinning mode of soliton,  $i\vec{\tau} \cdot \vec{K} = A^+\dot{A}$ . This leads to the time-dependent Lagrange function

$$\mathcal{L}(t) = \int d\vec{r}\mathcal{L} = -M_H(\Theta, G, \omega, \sigma) + \Lambda(\Theta, G, \omega, \sigma, \phi, \xi_1, \xi_2)\text{Tr}(\dot{A}\dot{A}^\dagger) . \tag{4}$$

Minimizing the classical mass  $M_H(\Theta, G, \omega, \sigma)$  leads to the coupled differential equations for  $\Theta, G, \omega$  and  $\sigma$  subject to the aforementioned boundary conditions. In the spirit of the large  $N_c$ -expansion, one then extremizes the moment of inertia  $\Lambda(\Theta, G, \omega, \sigma, \phi, \xi_1, \xi_2)$  which gives the coupled differential equations for  $\xi_1, \xi_2$  and  $\phi$  in the presence of the background profiles  $\Theta, G, \omega$  and  $\sigma$ . The pertinent boundary conditions are  $\phi(0) = \phi(\infty) = 0$ ,  $\xi'_1(0) = \xi_1(\infty) = 0$ ,  $\xi'_2(0) = \xi_2(\infty) = 0$ ,  $2\xi_1(0) + \xi_2(0) = 2$ . The masses of nucleon  $M_N$  and the mass of  $\Delta$ ,  $M_\Delta$ , are then given by  $M_N = M_H + 3/8\Lambda$  and  $M_\Delta = M_H + 3/15\Lambda$ .

The electromagnetic form factors are obtained in the usual way [8] are:

$$\begin{aligned}
G_E^S(\vec{q}^2) &= -\frac{4\pi m^2}{3g} \int_0^\infty j_0(qr)\omega(r)e^{-2\sigma/d}r^2dr, \\
G_M^S(\vec{q}^2) &= -\frac{2\pi M_N m^2}{3g\Lambda} \int_0^\infty \frac{j_1(qr)}{qr}\phi(r)e^{-2\sigma/d}r^2dr, \\
G_E^V(\vec{q}^2) &= \frac{4\pi}{\Lambda} \int_0^\infty j_0(qr) \left\{ \frac{f_\pi^2}{3}[4s_2^4 + (1+2c)\xi_1 + \xi_2]e^{-2\sigma/d} + \frac{g\phi\Theta's^2}{8\pi^2r^2} \right\} r^2dr, \\
G_M^V(\vec{q}^2) &= \frac{8\pi M_N}{3} \int_0^\infty \frac{j_1(qr)}{qr} \left\{ 2f_\pi^2[2s_2^4 - Gc]e^{-2\sigma/d} + \frac{3g}{8\pi^2}\omega\Theta's^2 \right\} r^2dr,
\end{aligned} \tag{5}$$

where  $s = \sin(\Theta)$ ,  $c = \cos(\Theta)$  and  $s_2 = \sin(\Theta/2)$ . The normalization is  $G_E^S(0) = G_E^V(0) = 1/2$ . Similarly, meson-nucleon vertex form factors may be calculated [11]. Of course, such strong interaction form factors are model-dependent quantities. In the soliton approach, however, they arise naturally as Fourier transforms of the meson distribution within the extended nucleon. Stated differently, the soliton acts as a source of an extended meson cloud, which leads to a meson-nucleon interaction region of finite extension. In momentum space, this extension can be interpreted as a corresponding form factor. We mention that such a picture underlies the inclusion of strong form factors say in OBE models. Coming back to our approach, these form

factors are most easily evaluated in the Breit-frame:

$$\begin{aligned}
G^\pi(-\vec{q}^2) &= \frac{8\pi M_N f_\pi}{3q} (\vec{q}^2 + m_\pi^2) \int_0^\infty j_1(qr) \sin(\Theta) r^2 dr = \\
&= \frac{8\pi M_N f_\pi}{3} \int_0^\infty \frac{j_1(qr)}{qr} \left[ -2\Theta'c - \Theta''rc + \Theta'^2rs + \frac{2s}{r} + rm_\pi^2s \right] r^2 dr, \\
G_E^\rho(-\vec{q}^2) &= \frac{2\pi}{g\Lambda} \int_0^\infty j_0(qr) \left[ -\xi_1'' - \frac{2\xi_1'}{r} + m^2\xi_1 - \frac{\xi_2''}{3} - \frac{2\xi_2'}{3r} + \frac{m^2\xi_2^2}{3} \right] r^2 dr, \\
G_M^\rho(-\vec{q}^2) &= -\frac{8\pi M_N}{3g} \int_0^\infty \frac{j_1(qr)}{qr} \left[ -G'' + 2G/r^2 + m^2G \right] r^2 dr, \\
G_E^\omega(-\vec{q}^2) &= 4\pi \int_0^\infty j_0(qr) \left[ \omega'' + \frac{2\omega'}{r} - m^2\omega \right] r^2 dr, \\
G_M^\omega(-\vec{q}^2) &= \frac{2\pi M_N}{\Lambda} \int_0^\infty \frac{j_1(qr)}{qr} \left[ \phi'' - \frac{2\phi}{r^2} - m^2\phi \right] r^2 dr,
\end{aligned} \tag{6}$$

where the “electric” and “magnetic” vector meson–nucleon form factors are connected to the Dirac  $F_1(t)$  and Pauli form factors  $F_2(t)$  through the following relations:  $G_E^i(t) = F_1^i(t) + tF_2^i(t)/4M_N^2$ ,  $G_M^i(t) = F_1^i(t) + F_2^i(t)$ , ( $i = \rho, \omega$ ).

### 3 Results and discussions.

#### 3.1 Static and electromagnetic properties of the nucleon.

Using the formulas given above we have calculated static and electromagnetic properties of nucleon. As can be seen from Eq.(1), the Lagrangian has no free parameters in the  $\pi\rho\omega$  sector. So, in actual calculations the parameters  $m_\pi, m, f_\pi$  are fixed at their emperical values,  $m_\pi = 138$  MeV,  $m = m_\rho = m_\omega = 770$  MeV,  $f_\pi = 93$  MeV,  $g = m/\sqrt{2}f_\pi = 5.85$ . In the  $\sigma$ -meson sector there are in general three free parameters:  $m_\sigma, S_0$  and the scale dimension  $d$ . The latter has been well studied for nuclear matter calculations [1, 2]. In particular, it was shown that for  $d \geq 2$  the much debated Brown–Rho (BR) scaling may be recovered. Therefore, assuming that there is no dependence of  $d$  on the density, we shall use the best value  $d = 2.6$  found in refs. [1, 2]. The values for  $S_0$  - were found to be  $S_0 = 90.6 \div 95.6$  MeV [1]. So we put  $S_0 = f_\pi = 93$  MeV. The mass of the  $\sigma$  or equivalently the gluon condensate  $C_g = m_\sigma^2 d^2 S_0^2 / 4 = m_\sigma^2 d^2 f_\pi^2 / 4$  is uncertain. We thus consider two cases:  $m_\sigma = 550$  MeV and  $m_\sigma = 720$  MeV in accordance with recent  $\pi\pi$  phase shift analyses [5] and with OBE values. We stress again that the precise nature of such a scalar–isoscalar field is not relevant here, only that it should not be a pure gluonium state. A summary of static nucleon properties obtained in both cases, i.e. with  $m_\sigma = 550$  MeV and  $m_\sigma = 720$  MeV, is given in Table 1. One immediately observes that the nucleon mass is again overestimated. This may not be regarded as a deficiency, since it is known that quantum fluctuations tend to decrease the mass substantially.

To estimate the influence of the  $\sigma$ -meson we also show the results given by minimal version of  $\pi\rho\omega$  model. As can be seen from Table 1, the inclusion of a light sigma meson into the basic

$\pi\rho\omega$  model just slightly changes the nucleon mass and its electromagnetic properties. This may be explained by the fact that the role of intermediate scalar–isoscalar meson in gamma–nucleon interactions is negligible. In contrast, the presence of the sigma–meson leads to an enhancement of axial coupling constant as it was first observed in ref. [12]. Although, the physical mechanism of this change is not clear, it may be understood as mainly due to modification of meson profiles. This is in marked contrast to the inclusion of pion loop effects, which tend to lower the axial coupling even further [13]. In the present model  $g_A = 0.88$  for the  $\pi\rho\omega$  and  $g_A = 0.95$  for the  $\pi\rho\omega\sigma$  model, respectively. One may expect that an appropriate inclusion of  $\sigma$  meson into a more complete version of the basic  $\pi\rho\omega$  model might give the desired value  $g_A = 1.26$ .

### 3.2 Meson–nucleon form factors and NN interaction

One of the usual ways to calculate the meson–nucleon interaction potential within topological soliton models is the so–called product ansatz [7]. Within this approximation, the two–Skyrmion potential as a function of the relative angles of orientation between the Skyrmions has a compact form, and the extraction of the NN potential by projection onto asymptotic two–nucleon states is straightforward. This procedure gives only three nonvanishing channels: the central, spin–spin and tensor potential. At large and intermediate distances, the latter two compare well with e.g. the phenomenological Paris potential [14]. The major inadequacy found in such type of calculations is the lack of an intermediate range attraction in central potential. Although many remedies have been proposed, this result may not be genuine for Skyrme like models. In fact, the product ansatz, which is not a solution to the equations of motion, can only be considered accurate at large distances, and the failure of these calculations to reproduce the central range attraction may simply be the failure of the product approximation to provide an adequate approximation to the exact solution.<sup>7</sup> Although the lack of central attraction may be recovered by the inclusion of a scalar–isoscalar meson, the inherent ansatz dependence of the trial configuration remains as a major shortcoming of product approximation [16].

On the other hand there is another natural way which was first used by Holzwarth and Machleidt [17]. They proposed to calculate  $V_{NN}$  within OBE model taking coupling constants and meson–nucleon form factors from a microscopical model such as the Cloudy Bag model or the Skyrme model. It was shown that the Skyrme form factor is a soft pion form factor that is compatible with the  $\pi N$  and  $NN$  systems. We shall use this strategy to investigate the  $NN$  potential within present model.

The meson nucleon form factors given by well known procedure, proposed first by Cohen [11] are given in Eq.(6). Although they were derived in a microscopical and consequent way, these form factors could not be directly used in standard OBE schemes. The reason is that the OBE schemes [6] in momentum space use form factors defined for fields propagating on a flat metric, whereas the definition of form factors in Eq.(6) involve a nontrivial metric. Hence, before using the latters in OBE scheme one should modify the procedure by redefining meson fields. The modification for pion nucleon form factors in  $\pi\rho\omega$  model is clearly outlined in refs. [17]. Now,

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<sup>7</sup>An early study giving credit to this line of reasoning can be found in ref.[15].

applying this procedure in the lagrangian in Eq. (1) we get the following  $\pi NN$  form factor:

$$\begin{aligned} G^\pi(-\vec{q}^2) &= \frac{8\pi M_N f_\pi}{3q} (\vec{q}^2 + m_\pi^2) \int_0^\infty j_1(qr) \sqrt{M_T(r)} \sin(\Theta) r^2 dr = \\ &= \frac{8\pi M_N f_\pi}{3} \int_0^\infty \frac{j_1(qr)}{qr} \left[ -2F'(r) - rF''(r) + \frac{2F(r)}{r} + r m_\pi^2 F(r) \right] r^2 dr, \end{aligned} \quad (7)$$

where

$$\begin{aligned} M_T &= [1 + 2 \tan^2(\Theta/2)] e^{-2\sigma/d}, \\ F(r) &= \sqrt{3 + \cos^2(\Theta(r)) - 4 \cos(\Theta(r))} e^{-\sigma/d}. \end{aligned} \quad (8)$$

The influence of metric factor  $M_T$  to the pion–nucleon form factor is illustrated in Fig. 1. It is seen that, without the inclusion of  $M_T$  the form factor is softer than in OBE models (the dashed line in Fig. 1), while its inclusion via Eqs. (7), (8) gives a behavior closer to OBE models. In fact, a monopole approximation at small  $q^2 = t$  of the normalised form factor  $G^\pi(t)/G^\pi(0) \approx \Lambda_\pi^2/(\Lambda_\pi^2 - t)$  gives  $\Lambda_\pi = 860$  MeV and  $\Lambda_\pi = 1100$  MeV for  $M_T = 1$  and  $M_T \neq 1$  respectively, compared to its empirical fit :  $\Lambda_\pi^{OBE} = 1300$  MeV (dotted line in Fig.1). Note, however, that our results for  $\Lambda_\pi$  are in line with recent coupled–channel calculations of the Jülich group [18]. There, a monopole form factor with  $\Lambda_\pi \simeq 800$  MeV is obtained. We do not want to stress here any qualitative comparison but rather like to point out that our approach also leads to cut–off values well below the ones obtained in OBE approaches.

Introducing a flat metric requires a canonical form for the kinetic part of the lagrangian, which determines the dynamics of the field fluctuation. The kinetic term of the scalar meson in Eq. (1)  $\mathcal{L}_\sigma^{\text{kin}} = S_0^2 e^{-2\sigma/d} \partial_\mu \sigma \partial^\mu \sigma / 2$  can be easily rewritten in a usual way:  $\mathcal{L}_\sigma^{\text{kin}} = \partial_\mu \tilde{\sigma} \partial^\mu \tilde{\sigma} / 2$  by the following redefinition of the basic sigma field:  $\tilde{\sigma}(r) = S_0 d [1 - e^{-\sigma(r)/d}]$ . Now the new field  $\tilde{\sigma}$  may be identified with the real sigma field. Clearly this redefinition does not change the nucleons static properties given in Table 1. Note also that, using the above redinition in the last term of Eq. (1), one may easily conclude that  $m_{\tilde{\sigma}} = m_\sigma$ . The appropriate sigma–nucleon form factor is given by

$$G^\sigma(-\vec{q}^2) = -4\pi \int_0^\infty j_0(qr) \left[ \tilde{\sigma}'' + \frac{2\tilde{\sigma}'}{r} - m_\sigma^2 \tilde{\sigma} \right] r^2 dr, \quad (9)$$

and may be used in OBE models. We have not introduced any metric factors in the form factors of the heavier mesons since these should play a lesser role than in the case of the pion.

For small values of the squared four–momentum transfer  $t$  each form factor can be parametrized in monopole form:  $G_i(t) = g_i(\Lambda_i^2 - m_i^2)/(\Lambda_i^2 - m_i^2)$  ( $i = \pi, \rho, \omega, \sigma$ ). We present in Table 2 the range parameters (cut–offs) and the coupling constants of the resulting meson–nucleon dynamics. One can see that there is no interference between  $\sigma$ –meson nucleon and e. g. the pion–nucleon coupling constants. In other words, the inclusion of  $\sigma$ –meson does not significantly affect meson–nucleon form factors that had been given by the  $\pi\rho\omega$  model. As it is seen from Table 2 the values for meson–nucleon coupling constants are close to their empirical values (in some cases obtained by OBE model fits). This is one of the main advantages of the inclusion of a scalar–isoscalar meson as done in the present approach.

The corollary of the present model is that it gives significant information on the  $\sigma$ –nucleon interaction. As it is seen from the Table 2, the value for  $g_{\sigma NN}$  and the cut–off parameter of

sigma–nucleon vertex  $\Lambda_\sigma$  are smaller than their OBE prediction  $\Lambda_\sigma^{OBE} \approx 1300 \div 2000$  MeV. This contrast is evidently seen from the Fig. 2., where  $G^\sigma(t)/G^\sigma(0)$  for two cases :  $m_\sigma = 720$  MeV and  $m_\sigma = 550$  MeV is presented with the solid and dashed lines respectively. The band enclosed by the dotted lines refers to the OBE monopole form factor with  $\Lambda_\sigma^{OBE} = 1300 \dots 2000$  MeV. One can conclude that the present model gives a softer  $\sigma NN$  form factor than it obtained by OBE. As it had been noticed before, the  $t$ –plane for each form factor has a cut along the positive real axis extending from  $t = t_0$  to  $\infty$ . The cut for the  $\sigma$ –nucleon vertex function starts at  $t_0 = 4m_\pi^2$  reflecting the kinematical threshold for the  $\sigma \rightarrow \pi\pi$  channel. More precisely this result follows from the asymptotic behavior of the meson profiles: For  $r \rightarrow \infty$ , we have  $\Theta(r) \sim \exp(-m_\pi r)/m_\pi r$ ,  $\sigma(r) \sim \Theta^2(r)$ , which are derived from the equations of motion.

Once the vertex function of the corresponding meson–nucleon interaction has been found, its appropriate contribution to the  $NN$  interaction may be easily calculated by using well known techniques from OBE. The detailed formulas are given elsewhere [6, 19]. In particular, the contribution of the  $\sigma$ –meson exchange to the central potential is given by

$$V_\sigma^c(r) = \int_0^\infty \frac{k^2 dk}{2\pi^2} \frac{G_{\sigma NN}^2(k^2)}{k^2 + m_\sigma^2} j_0(kr) . \quad (10)$$

The central  $NN$  potential in the  $T = 0, S = 1$  state (the deuteron state) is presented in Fig. 3 in comparison with Paris potential. Our prediction is in good agreement with the empirical one. Note that the desired attraction in the central  $V_{NN}$  has before been obtained in the  $\pi\rho\omega$  model by means of two–meson exchange [19].

In conclusion it should be noted that we do not intend (expect) to describe (cover) all  $NN$  phase shifts staying only in the framework of the present model. Besides other mesons, which are usually included in OBE picture, the full model should also take in account e.g.  $N\Delta\rho$  couplings. In addition the  $2\pi$  exchange and its strong mixing with  $\sigma$  meson exchange (see e.g. ref.[20]) should be considered. Another reason possible which limits the accuracy of  $NN$  phase shifts in the present model is that the  $\sigma NN$  coupling is not sensitive the mass of sigma (see Table 2) as it is in the OBE phenomenology. In fact, even when the  $2\pi$  exchange is disregarded, the pure OBE model has to consider two types of sigmas with nearly the same masses but with quite different coupling constants. So, we refrain from performing direct calculations of  $NN$  phase shifts in the present model. Instead, we point out that the meson–nucleon form factors found in the present model could be useful in a wider context of calculations of nucleon–nucleon observables (phase shifts, deuteron properties etc) and may give more information on meson–nucleon and nucleon–nucleon dynamics.

To summarize, we have developed a topological chiral soliton model with an explicit light scalar–isoscalar meson field, which plays a central role in nuclear physics, based on the chiral symmetry and broken scale invariance of QCD. We have shown that for the single nucleon properties, the successful description of the electromagnetic observables of the  $\pi\rho\omega$  model is not modified and even the value for the axial–vector coupling is somewhat improved. In the two–nucleon sector, this extended  $\pi\rho\omega\sigma$  Lagrangian leads to the correct intermediate range attraction in the central potential and a soft  $\sigma NN$  formfactor for both values of sigma meson mass  $m_\sigma = 550$  MeV and  $m_\sigma = 720$  MeV.

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Table 1: Baryon properties in the  $\pi\rho\omega$  and  $\pi\rho\omega\sigma$  models

	$\pi\rho\omega$	$\pi\rho\omega\sigma$	$\pi\rho\omega\sigma$	Exp.
$m_\sigma$ [MeV]	—	550	720	
$C_g^{1/4}$ [MeV]	—	258	295	$300\div 400$
$B^{1/4}$ [MeV]	—	119	121	—
$M_N$ [MeV]	1560	1492	1511	939
$\Lambda$ [fm]	0.88	0.88	0.88	—
$M_\Delta - M_N$ [MeV]	344	350	350	293
$r_H = \langle r_B^2 \rangle^{1/2}$ [fm]	0.5	0.5	0.5	$\sim 0.5$
$\langle r_E^2 \rangle_p^{1/2}$ [fm]	0.92	0.94	0.94	$0.86 \pm 0.01$
$\langle r_E^2 \rangle_n$ [fm <sup>2</sup> ]	-0.20	-0.16	-0.16	$-0.119 \pm 0.004$
$\langle r_M^2 \rangle_p^{1/2}$ [fm]	0.84	0.85	0.85	$0.86 \pm 0.06$
$\langle r_M^2 \rangle_n^{1/2}$ [fm]	0.85	0.85	0.85	$0.88 \pm 0.07$
$\mu_p$ [n.m.]	3.34	3.33	3.33	2.79
$\mu_n$ [n.m.]	-2.58	-2.53	-2.53	-1.91
$ \mu_p/\mu_n $	1.29	1.30	1.30	1.46
$g_A$	0.88	0.95	0.95	$1.26 \pm 0.006$
$\langle r_A^2 \rangle_p^{1/2}$ [fm]	0.63	0.66	0.66	$0.65 \pm 0.07$

Table 2: Meson–nucleon coupling constants and cut–off parameters of meson–nucleon form factors. The  $\Lambda_i (i = \pi, \rho, \omega\sigma)$  are cutoff parameters in equivalent monopole fits  $1/(1 - t/\Lambda_i^2)$  to the normalized form factors  $G_{iNN}(t)/G_{iNN}(0)$  around  $t = 0$ . The empirical values are from OBE potential fit [6]

	$\pi\rho\omega$	$\pi\rho\omega\sigma(550)$	$\pi\rho\omega\sigma(720)$	OBE/Emp.
$G_{\pi NN}(0)$	14.74	13.97	14.17	13.53
$G_{\sigma NN}(0)$	—	6.2	6.19	9.1 (12.41)
$F_1^\rho(0)$	2.55	2.76	2.68	2.24
$F_2^\rho(0)$	14.33	15.01	14.67	13.7
$F_2^\rho(0)/F_1^\rho(0)$	5.6	5.43	5.47	6.1
$F_1^\omega(0)$	8.78	10.73	10.15	11.7
$F_2^\omega(0)$	−2.15	−2.78	−2.65	0
$F_2^\omega(0)/F_1^\omega(0)$	−0.24	−0.25	−0.26	0
$\Lambda_\pi$ (GeV)	1.2	1.1	1.1	$1.3 \div 2.0$
$\Lambda_\sigma$ (GeV)	—	0.59	0.60	$1.3 \div 2.0$
$\Lambda_1^\rho$ (GeV)	0.62	0.63	0.63	1.3
$\Lambda_2^\rho$ (GeV)	0.92	0.92	0.92	1.3
$\Lambda_1^\omega$ (GeV)	0.95	0.89	0.91	1.5
$\Lambda_2^\omega$ (GeV)	1.12	0.86	0.89	—

Fig.1.

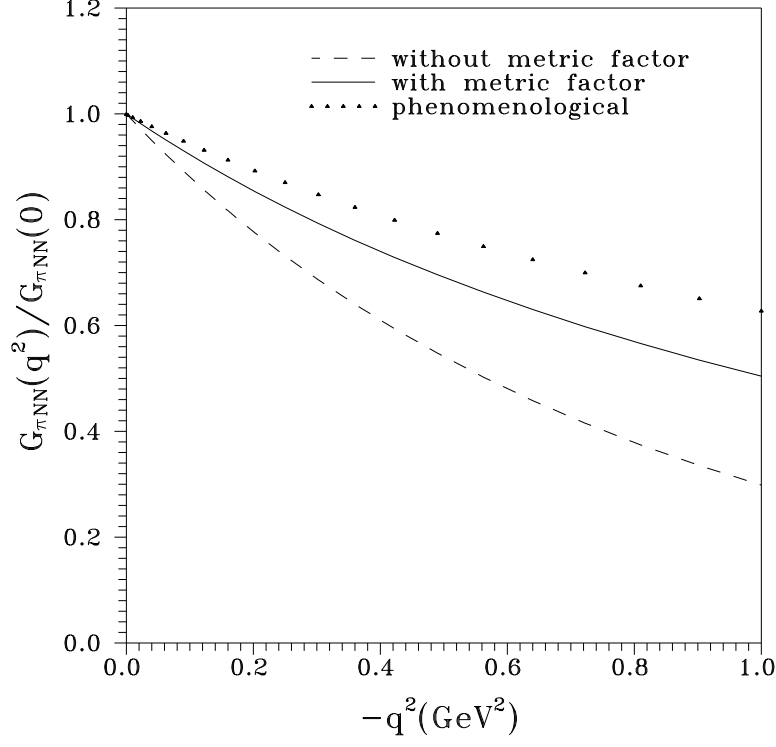


Figure 1: The normalized  $\pi NN$  form factor in the  $\pi\rho\omega\sigma$  model ( $m_\sigma = 720$  MeV). The solid line represents the form factor when the metric factor is included (Eq. (7)), while the dashed line gives the result with no metric factor as in Eq. (6). The dotted line is a monopole form factor with  $\Lambda_\pi = 1300$  MeV.

Fig.2.

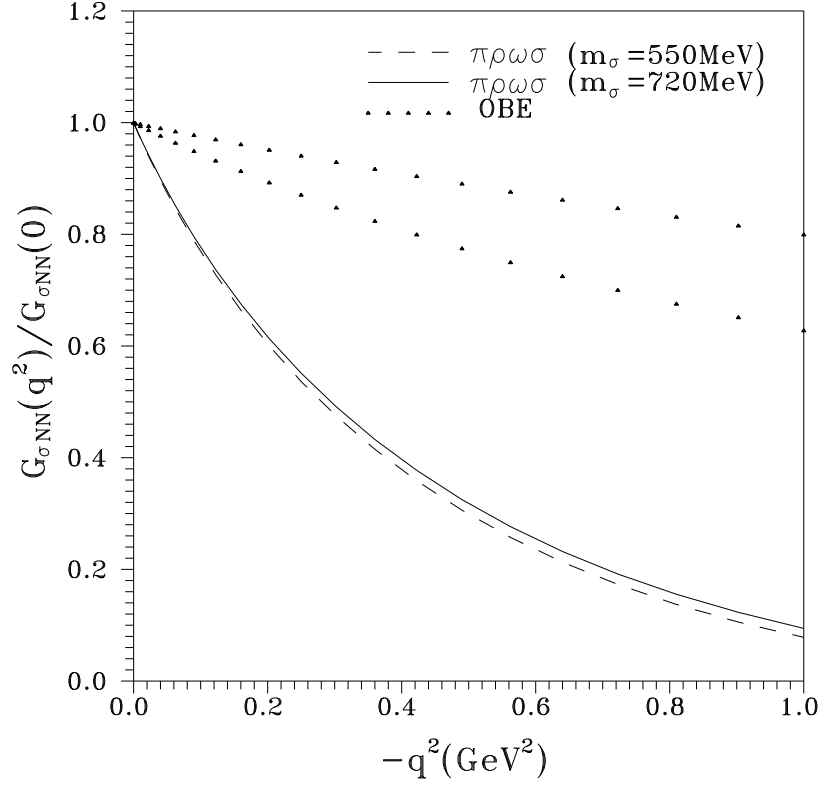


Figure 2: The sigma-nucleon form factor  $G_{\sigma NN}(\vec{q}^2)/G_{\sigma NN}(0)$ . The dashed and solid lines are for  $m_\sigma = 550 \text{ MeV}$  and  $m_\sigma = 720 \text{ MeV}$ , respectively. Typical OBE monopole fits with  $\Lambda = 1.3 \dots 2 \text{ GeV}$  are shown by the band enclosed by the dotted lines.

Fig.3.

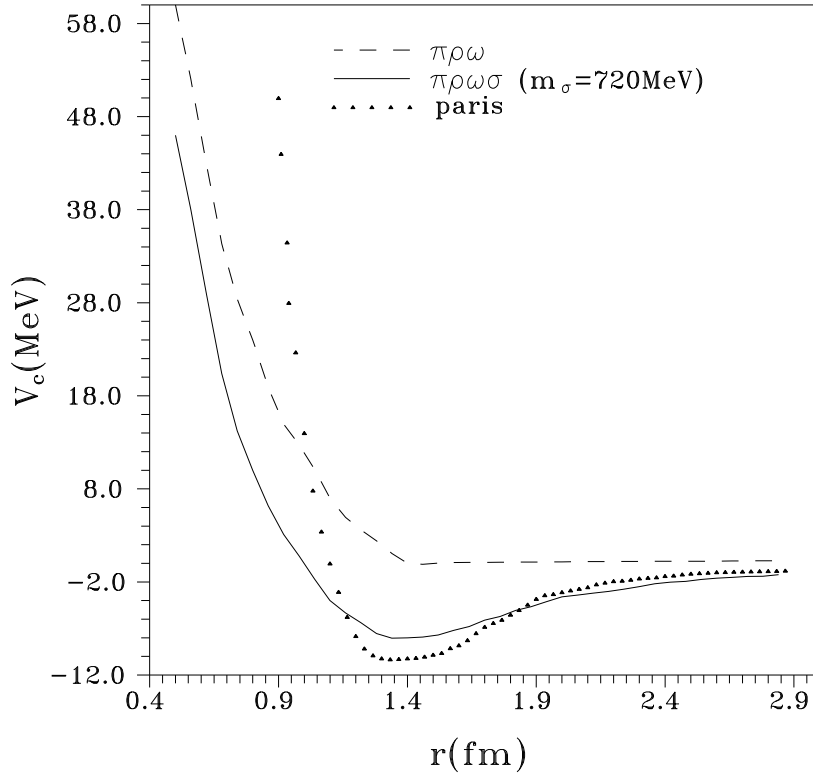


Figure 3: The central potential in the  $S = 1$ ,  $T = 0$  state for  $\pi\rho\omega$  and  $\pi\rho\omega\sigma$  models (dashed and solid lines, respectively). No contribution from two-meson exchange has been taken into account. The dotted line corresponds to the Paris potential [14].